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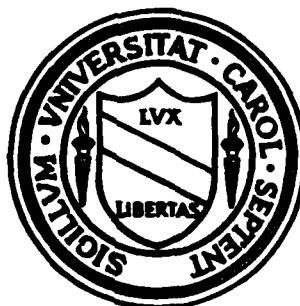
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CENTER FOR STOCHASTIC PROCESSES

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ADMISSIBLE TRANSLATES OF STABLE PROCESSES :
A SURVEY AND SOME NEW MODELS

by

Stamatis Cambanis

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ADMISSIBLE TRANSLATES OF STABLE PROCESSES:
A SURVEY AND SOME NEW MODELS

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ABSTRACT

→ This document

We survey some recent results on the admissible translates of stable processes and we contrast them with the analogs for Gaussian processes. Whereas Gaussian moving averages and Fourier transforms of independent increments processes have rich classes of admissible translates, their stable counterparts frequently have all translates singular. By removing the requirement of independence of the increments, we introduce stable processes that are generalized moving averages and harmonizable which can have rich classes of admissible translates. These are generally nonstationary processes but we also show a class of stationary generalized moving averages. (See page 1)

1. INTRODUCTION

Throughout $X = \{X(t, \omega), -\infty < t < \infty\}$ is a stochastic process defined on a probability space (Ω, \mathcal{F}, P) , and $s = \{s(t), -\infty < t < \infty\}$ is a nonrandom function. The map $\omega \rightarrow X(\cdot, \omega)$ from the probability space to the space of all functions on the real line, induces a probability measure P_X on the σ -field of cylinder sets, the distribution of the process X . If the distribution of the translate of X by s , P_{s+X} , is absolutely continuous with respect to the distribution of X , P_X , then s is called an admissible translate of X , and if P_{s+X} and P_X are singular, then s is called a singular translate. The set of all admissible translates of X is denoted by $AT(X)$.

In signal detection s is the signal, X is the random noise, and based on a sample observation (a function on the real line) one decides whether the observation is due to signal plus noise ($s+X$) or to noise alone (X). For a singular translate s , in principle, a correct decision can be made with probability one, i.e. signals that are singular translates of the noise can be perfectly detected. For an admissible translate s the decision is based on the Radon-Nikodym derivative of P_{s+X} with respect to P_X (likelihood ratio) according to the Neyman-Pearson rule and the resulting probability of detecting correctly the presence of the signal is always strictly less than one. Most real life signal detection problems correspond to admissible translates.

In Section 2 we describe the well-known complete results on the admissible and singular translates of Gaussian processes. In Section 3 we summarize some recent results for stable processes which are far from being complete. We refer to [1] and references therein.

It turns out that the widely used moving average and Fourier transform models typically have all translates singular in the non-Gaussian stable case, and thus do not provide realistic noise models for signal detection. We thus introduce in Section 3 some generalized moving averages and harmonizable processes which in the non-Gaussian stable case may have rich classes of admissible translates and therefore serve as stable noise models in signal detection.

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2. GAUSSIAN PROCESSES

Let X be a Gaussian process with mean zero and covariance function R . The set of admissible translates of X is precisely the reproducing kernel Hilbert space of its kernel R , and all other translates are singular. More concrete representations follow.

Every Gaussian process X can be represented in terms of a Gaussian process ξ with independent increments as follows:

$$(1) \quad X(t) = \int_{-\infty}^{\infty} f(t, \lambda) d\xi(\lambda), \quad -\infty < t < \infty,$$

where $f(t, \cdot) \in L_2(\mu)$ and μ is the control measure of ξ : $d\mu(\lambda) = E |d\xi(\lambda)|^2$. The integral $\int g d\xi$ is defined for all $g \in L_2(\mu)$ and is a normal random variable with characteristic function

$$E \exp\{ir \int g d\xi\} = \exp\{-\frac{1}{2} r^2 \int |g|^2 d\mu\}.$$

The covariance function of X is represented as $R(t, s) = \int_{-\infty}^{\infty} f(t, \lambda) \bar{f}(s, \lambda) d\mu(\lambda)$. Then

$$(2) \quad AT(X) = \{s(t) = \int_{-\infty}^{\infty} f(t, \lambda) g(\lambda) d\mu(\lambda), \quad g \in L_2(\mu)\}.$$

Stationary Gaussian processes (that are continuous in probability) have a spectral representation:

$$(3) \quad X(t) = \int_{-\infty}^{\infty} e^{it\lambda} d\xi(\lambda),$$

where ξ has independent increments and finite spectral (control) measure μ , and their covariance function is $R(t, s) = \int_{-\infty}^{\infty} e^{i(t-s)\lambda} d\mu(\lambda)$. Their admissible translates are Fourier transforms of finite signed measures that are absolutely continuous with respect to μ with μ -square integrable Radon-Nikodym derivative:

$$(4) \quad AT(X) = \{s(t) = \int_{-\infty}^{\infty} e^{it\lambda} d\nu(\lambda), \quad \nu \ll \mu, \quad \frac{d\nu}{d\mu} \in L_2(\mu)\}.$$

When $\mu \ll \text{Leb}$ with spectral density $\phi(\lambda) = d\mu(\lambda)/d\lambda$ then

$$AT(X) = \{s(t) = \int_{-\infty}^{\infty} e^{it\lambda} S(\lambda) d\lambda, \quad \int_{-\infty}^{\infty} \frac{|S(\lambda)|^2}{\phi(\lambda)} d\lambda < \infty\},$$

and in addition to the spectral representation (3), X also has a moving average representation:

$$(5) \quad X(t) = \int_{-\infty}^{\infty} h(t-s) d\xi(s),$$

where in (5), ξ has stationary independent increments (i.e., $\mu = \text{Leb}$) and $h \in L_2$; its covariance function is $R(t,s) = \int_{-\infty}^{\infty} e^{i(t-s)\lambda} |\hat{h}(\lambda)|^2 d\lambda$; and its admissible translates are

$$(6) \quad AT(X) = \{ s(t) = \int_{-\infty}^{\infty} h(t-s) g(s) ds, \quad g \in L_2 \}.$$

For nonstationary processes representations in terms of processes with dependent increments are useful. Every Gaussian process X can be represented in terms of a Gaussian process η with possibly dependent increments as follows:

$$(1') \quad X(t) = \int_{-\infty}^{\infty} f(t,u) d\eta(u), \quad -\infty < t < \infty.$$

Here η is an $L_2(\Omega, \mathcal{F}, P)$ -valued Gaussian measure generally not independently scattered, with bimeasure β : $\beta(B,C) = E\{ \eta(B) \bar{\eta}(C) \}$, B, C : bounded Borel sets, and each $f(t, \cdot)$ is η -integrable. The covariance of X is $R(t,s) = \iint_{-\infty}^{\infty} f(t,u) \bar{f}(s,v) d\beta(u,v)$, and its admissible translates are

$$(2') \quad AT(X) = \{ s(t) = \iint_{-\infty}^{\infty} f(t,u) g(v) d\beta(u,v) = \int_{-\infty}^{\infty} f(t,u) d\beta_g(u), \quad g : \eta\text{-integrable} \}$$

where β_g is the measure $\beta_g(\cdot) = \int_{-\infty}^{\infty} g(v) \beta(\cdot, dv)$.

When $f(t,u) = e^{itu}$ in (1') and η has bounded semi-variation, then X is harmonizable:

$$(3') \quad X(t) = \int_{-\infty}^{\infty} e^{itu} d\eta(u),$$

with bispectral measure β and covariance $R(t,s) = \iint_{-\infty}^{\infty} e^{i(tu-sv)} d\beta(u,v)$. It is stationary only if its bispectral measure β is concentrated on the diagonal of the plane. Its admissible translates are:

$$(4') \quad AT(X) = \{ s(t) = \int_{-\infty}^{\infty} e^{itu} d\beta_g(u), \quad g : \eta\text{-integrable} \}.$$

When $f(t,u) = h(t-u)$ in (1'), X is a "generalized" moving average (of a dependent increments

process):

$$(5') \quad X(t) = \int_{-\infty}^{\infty} h(t-u) d\eta(u),$$

whose admissible translates are

$$(6') \quad AT(X) = \{ s(t) = \int_{-\infty}^{\infty} h(t-u) d\beta_g(u), \quad g : \eta\text{-integrable} \}.$$

The representations of the admissible translates of general nonstationary processes, (2'), (4'), and (6'), are similar to those of stationary processes, (2), (4) and (6), but not as explicit and simple because generally no explicit description of the η -integrable functions is available.

3. STABLE PROCESSES

In this section X is a symmetric α -stable (S α S) process ($0 < \alpha \leq 2$), i.e. all linear combinations $c_1 X(t_1) + \dots + c_n X(t_n)$ are S α S random variables (with characteristic functions of the form $\exp\{-\text{const. } |r|^\alpha\}$). When $\alpha = 2$, X is Gaussian. When $0 < \alpha < 2$, X is non-Gaussian stable and can be represented as in (1), where ξ has S α S independent increments and control measure μ . Here the integral $\int g d\xi$ is defined for all $g \in L_\alpha(\mu)$ and is a S α S random variable with characteristic function

$$E \exp\{ir \int g d\xi\} = \exp\{-|r|^\alpha \int |g|^\alpha d\mu\}.$$

A crucial difference in the representation (1) between the Gaussian ($\alpha=2$) and non-Gaussian ($0 < \alpha < 2$) cases is that whereas in the Gaussian case the linear space of the increments of ξ can always be taken to be equal to the linear space of the process X , in the non-Gaussian case the former is generally larger than the latter (and generally infinite dimensional even when the linear space of X is finite dimensional).

The analog of (2) is no longer valid when $0 < \alpha < 2$ and all that can be said in general is that

$$(7) \quad AT(X) \subset \{ s(t) = \int_{-\infty}^{\infty} f(t, \lambda) g(\lambda) d\mu(\lambda), \quad g \in L_{\alpha*}(\mu) \} \triangleq F(X),$$

where $1/\alpha + 1/\alpha^* = 1$, and that translates outside the set $F(X)$ are singular. $AT(X)$ may be as large as $F(X)$ (e.g. when X is sub-Gaussian) or as small as $\{0\}$ (e.g. when X is Lévy motion).

In order to describe better the contrasts and similarities between the Gaussian and other stable cases we first concentrate on the independently scattered SαS measure $\xi = \{ \xi(B), B \in \mathcal{B}_b \}$ in (1), where \mathcal{B}_b denotes the bounded Borel sets of the real line. When $\alpha = 2$, $AT(\xi)$ consists of all signed measures which are absolutely continuous with respect to μ with Radon-Nikodym derivative in $L_2(\mu)$. In sharp contrast when $0 < \alpha < 2$ and μ is nonatomic, then ξ has no admissible translates and in fact all translates are singular. On the other hand, when $0 < \alpha < 2$ and μ is purely atomic with atoms $\{a_n\}_{n=1}^N$, $1 \leq N \leq \infty$, then $AT(\xi)$ consists of all measures s concentrated on $\{a_n\}_{n=1}^N$ with

$$\sum_{n=1}^N |s(\{a_n\})|^2 / \mu^{2/\alpha}(\{a_n\}) < \infty ;$$

this result is due to Shepp [2] and is valid for all $0 < \alpha \leq 2$. For a general control measure μ , $AT(\xi) = \{0\}$ when μ has no atoms, and when μ has atoms $\{a_n\}_{n=1}^N$ then $AT(\xi)$ is as above.

We now consider a general SαS process with representation (1) and $0 < \alpha < 2$. X is called invertible if its representors $\{f(t, \cdot), -\infty < t < \infty\}$ are complete in $L_\alpha(\mu)$, i.e. if the linear spaces of X and of the increments of ξ are equal. When X is invertible then every translate is either admissible or singular and $AT(X) = \{0\}$ when μ has no atoms and if μ has atoms $\{a_n\}_{n=1}^N$, $1 \leq N \leq \infty$, then

$$AT(X) = \{ s(t) = \sum_{n=1}^N s_n \mu^{1/\alpha}(\{a_n\}) f(t, a_n), \sum_{n=1}^N |s_n|^2 < \infty \} .$$

Here $AT(X)$ is a proper subset of $F(X)$ even when μ is purely atomic.

When X is stationary and has the spectral representation (3), where ξ has independent SαS increments and finite spectral (control) measure μ , then X is invertible. In the Gaussian case its admissible translates are described in (4). In the non-Gaussian case $0 < \alpha < 2$, if the spectral measure μ has no atoms all translates of X are singular and if μ has atoms $\{a_n\}_{n=1}^N$, $1 \leq N \leq \infty$, then

$$AT(X) = \{ s(t) = \sum_{n=1}^N s_n \mu^{1/\alpha}(\{a_n\}) e^{it a_n}, \sum_{n=1}^N |s_n|^2 < \infty \} .$$

When X is stationary and has a moving average representation (5), where ξ has stationary independent S α S increments, then all translates of X are singular when $0 < \alpha < 2$, provided the translates of its kernel h are complete in L_α . In the Gaussian case the admissible translates of X are described in (6).

The processes with spectral (3) and those with moving average (5) representation are the most widely studied classes of stationary stable processes, and are actually disjoint when $0 < \alpha < 2$, while in the Gaussian case $\alpha = 2$ the latter is a subset of the former. Since when $0 < \alpha < 2$ in most cases they have no admissible translates and all their translates are singular, they do not provide reasonable models of stable noise in signal detection, as every signal can be detected perfectly in principle in the presence of such noise. One way to introduce more realistic stable noise models is to allow the increments of ξ in (3) and (5) to have some dependence, i.e. to consider the stable counterparts of (3') and (5').

Thus let X have representation (1'), where η has dependent S α S increments with $0 < \alpha < 2$. Here η is a S α S measure which is not independently scattered and can be represented as

$$\eta(\cdot) = \int_{-\infty}^{\infty} \nu(\cdot, \lambda) d\xi(\lambda),$$

where ξ has independent S α S increments and control measure μ , and ν is an $L_\alpha(\mu)$ -valued measure. If g is ν -integrable then $\int g d\nu \in L_\alpha(\mu)$ and

$$\int_{-\infty}^{\infty} g d\eta = \int_{-\infty}^{\infty} \left\{ \int_{-\infty}^{\infty} g d\nu \right\} d\xi.$$

Thus with each $f(t, \cdot)$ being ν -integrable we have

$$(8) \quad X(t) = \int_{-\infty}^{\infty} f(t, u) d\eta(u) = \int_{-\infty}^{\infty} \left\{ \int_{-\infty}^{\infty} f(t, u) \nu(du, \lambda) \right\} d\xi(\lambda), \quad -\infty < t < \infty.$$

To ensure that η is not independently scattered we assume that for some disjoint Borel sets B_1 and B_2 , $\mu\{\lambda : \nu(B_1, \lambda) \nu(B_2, \lambda) \neq 0\} > 0$, since $\eta(B_1)$ and $\eta(B_2)$ are independent iff $\nu(B_1, \lambda) \nu(B_2, \lambda) = 0$ a.e. $[\mu]$. Note that $\eta(\{a\}) = 0$ iff $\int_{-\infty}^{\infty} |\nu(\{a\}, u)|^2 d\mu(u) = 0$, so that η may have no atoms even when ξ does, e.g. if ν has no atoms: $\nu(\{a\}, u) = 0$ a.e. $[\mu]$. It is therefore possible for X to have

admissible translates even when η has no atoms.

We further require X to be invertible, i.e. $\{ \int_{-\infty}^{\infty} f(t,u) \nu(du, \cdot), -\infty < t < \infty \}$ to be complete in $L_\alpha(\mu)$, and μ to have atoms $\{a_n\}_{n=1}^N, 1 \leq N \leq \infty$. It then follows that

$$AT(X) = \{ s(t) = \sum_{n=1}^N s_n \mu^{1/\alpha}(\{a_n\}) \int_{-\infty}^{\infty} f(t,u) \nu(du, a_n), \sum_{n=1}^N |s_n|^2 < \infty \}.$$

When $N < \infty$ the representation of an admissible translate can be written as in (2'): $s(t) = \int_{-\infty}^{\infty} f(t,u) d\nu_N(u)$ where the measure ν_N is

$$(9) \quad \nu_N(\cdot) = \sum_{n=1}^N s_n \mu^{1/\alpha}(\{a_n\}) \nu(\cdot, a_n).$$

When $N = \infty$ the infinite series in (9) does not generally define a measure and we can only write $s(t) = \lim_{k \rightarrow \infty} \int_{-\infty}^{\infty} f(t,u) d\nu_k(u)$, where ν_k is defined as in (9).

With $f(t,u) = e^{itu}$ in (8) and η of bounded semi-variation then X is harmonizable:

$$X(t) = \int_{-\infty}^{\infty} e^{itu} d\eta(u) = \int_{-\infty}^{\infty} \hat{\nu}(t, \lambda) d\xi(\lambda),$$

where $\hat{\nu}(t, \lambda) = \int_{-\infty}^{\infty} e^{itu} \nu(du, \lambda)$, with admissible translates all functions of the form (as in (4'))

$$s(t) = \int_{-\infty}^{\infty} e^{itu} d\nu_N(u) \text{ if } N < \infty, = \lim_{k \rightarrow \infty} \int_{-\infty}^{\infty} e^{itu} d\nu_k(u) \text{ if } N = \infty.$$

Harmonizable processes are nonstationary when η has dependent increments.

When $f(t,u) = h(t-u)$ in (8), X is a "generalized" moving average (of a dependent increments process):

$$(10) \quad X(t) = \int_{-\infty}^{\infty} h(t-u) d\eta(u) = \int_{-\infty}^{\infty} \left\{ \int_{-\infty}^{\infty} h(t-u) \nu(du, \lambda) \right\} d\xi(\lambda)$$

with admissible translates are all functions of the form (as in (6'))

$$(11) \quad s(t) = \int_{-\infty}^{\infty} h(t-u) d\nu_N(u) \text{ if } N < \infty, = \lim_{k \rightarrow \infty} \int_{-\infty}^{\infty} h(t-u) d\nu_k(u) \text{ if } N = \infty.$$

It is easy to construct examples of nonstationary generalized moving averages, but there exist sta-

tionary ones as well. Indeed, taking

$$\nu(B, \lambda) = \phi(\lambda) \int_B e^{ix\theta(\lambda)} dx$$

for bounded Borel sets B where $\phi \in L_\alpha(\mu)$ we have for $g \in L_1$, $\int_{-\infty}^{\infty} g(x) \nu(dx, \lambda) = \phi(\lambda) \hat{g}[\theta(\lambda)]$ and from (10),

$$X(t) = \int_{-\infty}^{\infty} e^{it\theta(\lambda)} \phi(\lambda) \hat{h}[-\theta(\lambda)] d\xi(\lambda).$$

Therefore X is stationary and invertible with admissible translates all functions of the form (as in (6) and (6'))

$$s(t) = \int_{-\infty}^{\infty} h(t-u) g(u) du,$$

where $g(u) = \sum_{n=1}^N s_n \mu^{1/\alpha}(\{a_n\}) \phi(a_n) e^{iu\theta(a_n)}$ and $\sum_{n=1}^N |s_n|^2 < \infty$.

Thus harmonizable and generalized moving averaged SαS processes can be constructed with rich classes of admissible translates which may serve as realistic models of noise in signal detection.

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